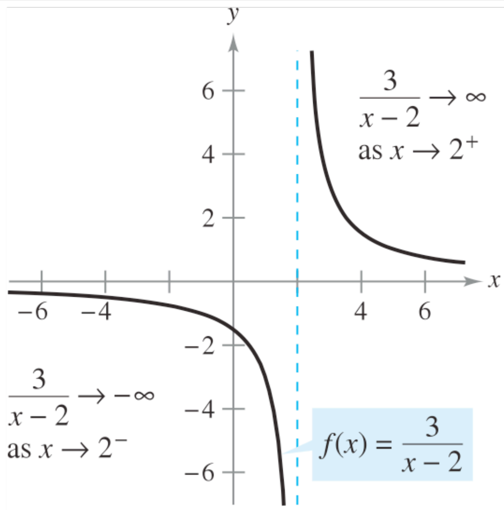
Math 150, *Lecture Notes- Bonds* Name____________

Section 1.5 Infinite Limits

Consider the function $f(x)=\frac{3}{x-2}$. In this case, we say that $f(x)$ ***decreases without bound as*** x ***approaches 2 from the left, and*** $f(x)$ ***increases without bound as*** x ***approaches 2 from the right***.



$f(x)$ increases and decreases without bound as x approaches 2.

x approaches 2 from the left. → ← x approaches 2 from the right.

x	1.5	1.9	1.99	1.999	2	2.001	2.01	2.1	2.5
$f(x)$	−6	−30	−300	−3000	?	3000	300	30	6

$f(x)$ decreases without bound. → ← $f(x)$ increases without bound.

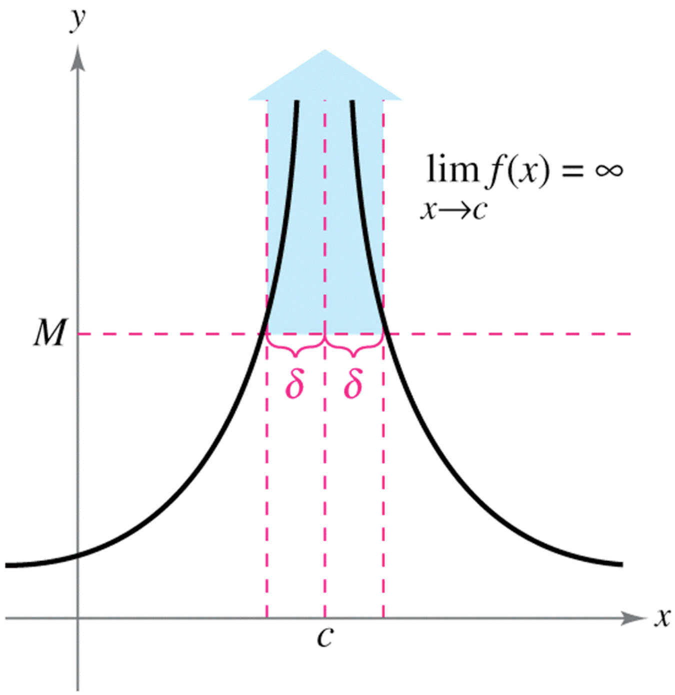
$$\lim_{x\to 2^-} \frac{3}{x-2} = -\infty$$

$f(x)$ decreases without bound as x approaches 2 from the left.

and

$$\lim_{x\to 2^+} \frac{3}{x-2} = \infty$$

$f(x)$ increases without bound as x approaches 2 from the right.



Definition of Infinite Limits

Let f be a function that is defined at every real number in some open interval containing c (except possibly at c itself). The statement

$$\lim_{x \to c} f(x) = \infty$$

means that for each $M > 0$ there exists a $\delta > 0$ such that $f(x) > M$ whenever $0 < |x - c| < \delta$ (see Figure 1.40). Similarly, the statement

$$\lim_{x \to c} f(x) = -\infty$$

means that for each $N < 0$ there exists a $\delta > 0$ such that $f(x) < N$ whenever $0 < |x - c| < \delta$. To define the **infinite limit from the left,** replace $0 < |x - c| < \delta$ by $c - \delta < x < c$. To define the **infinite limit from the right,** replace $0 < |x - c| < \delta$ by $c < x < c + \delta$.

The symbols ∞ and $-\infty$ do not represent real numbers. They are convenient symbols used to describe unbounded conditions more concisely.

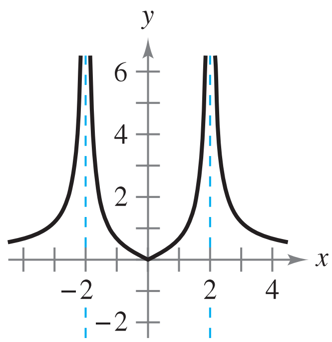
A limit in which $f(x)$ increases or decreases without bound as x approaches c is called an **infinite limit**.

Be sure that you see the equal sign in the statement $\lim f(x) = \infty$ does not mean that the limit exists! On the contrary, it tells us ***how*** the limit **fails to exist** by denoting the unbounded behavior of $f(x)$ as x approaches c.

Determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -2 from the left and from the right.

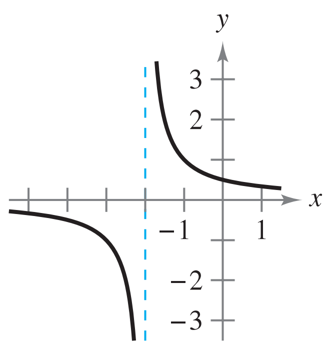
Ex.1

$$f(x) = 2\left|\frac{x}{x^2 - 4}\right|$$



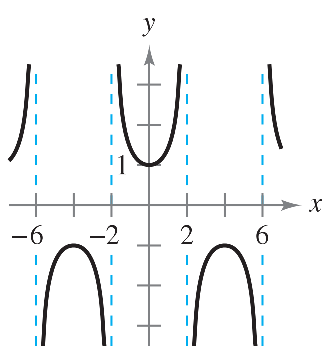
Ex.2

$$f(x) = \frac{1}{x + 2}$$



Ex.3

$$f(x) = \sec\frac{\pi x}{4}$$

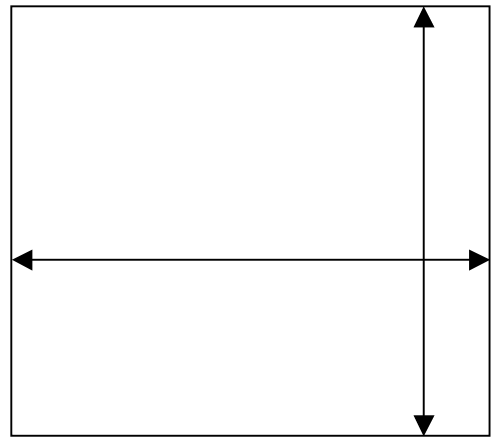


By completing the table, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -3 from the left and from the right. Graph the function to confirm your result.

Ex.4 $f(x)=\dfrac{x^2}{x^2-9}$

x	-3.5	-3.1	-3.01	-3.001
$f(x)$				

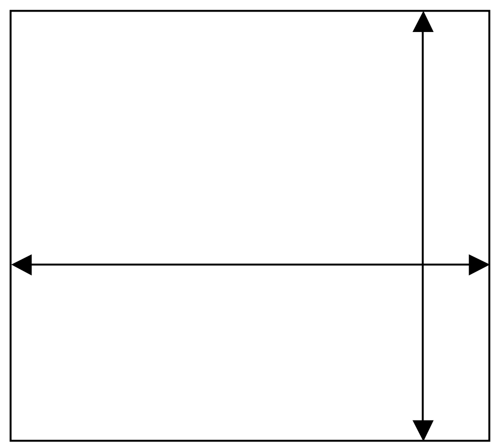
x	-2.999	-2.99	-2.9	-2.5
$f(x)$				



Ex.5 $f(x)=\cot\left(\dfrac{\pi x}{3}\right)$

x	-3.5	-3.1	-3.01	-3.001
$f(x)$				

x	-2.999	-2.99	-2.9	-2.5
$f(x)$				



Definition of Vertical Asymptote

If $f(x)$ approaches infinity (or negative infinity) as x approaches c from the right or the left, then the line $x = c$ is a **vertical asymptote** of the graph of f.

THEOREM 1.14 Vertical Asymptotes

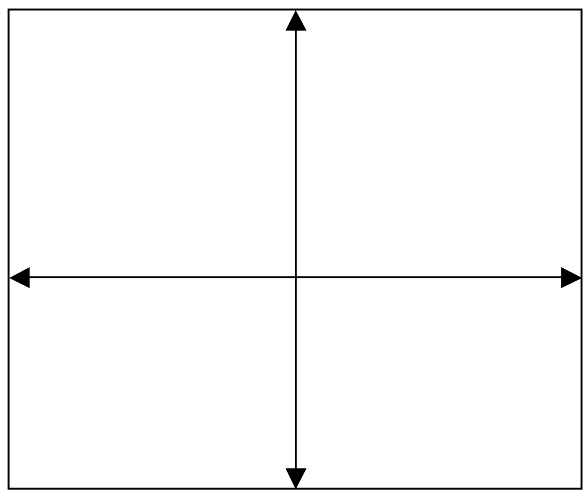
Let f and g be continuous on an open interval containing c. If $f(c) \neq 0$, $g(c) = 0$, and there exists an open interval containing c such that $g(x) \neq 0$ for all $x \neq c$ in the interval, then the graph of the function given by

$$h(x) = \frac{f(x)}{g(x)}$$

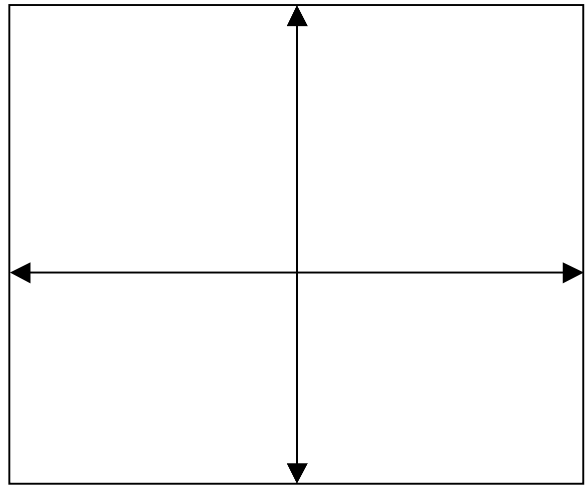
has a vertical asymptote at $x = c$.

Find the vertical asymptotes of the graph of the function. Graph the function to confirm your result.

Ex.6 $f(x) = \dfrac{x^2 - 9}{x^3 + 3x^2 - x - 3}$

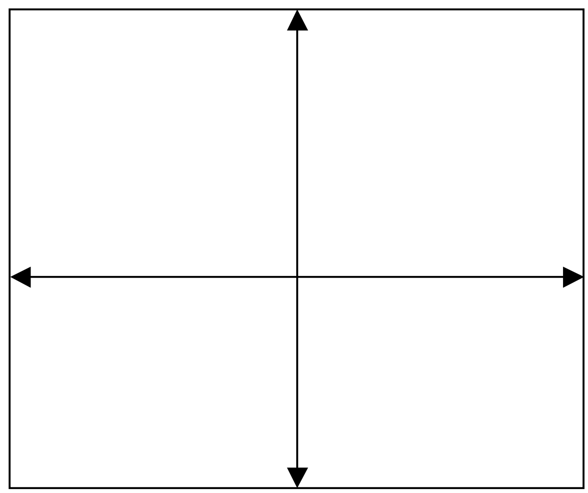


Ex.7 $f(x) = \tan(\pi x)$

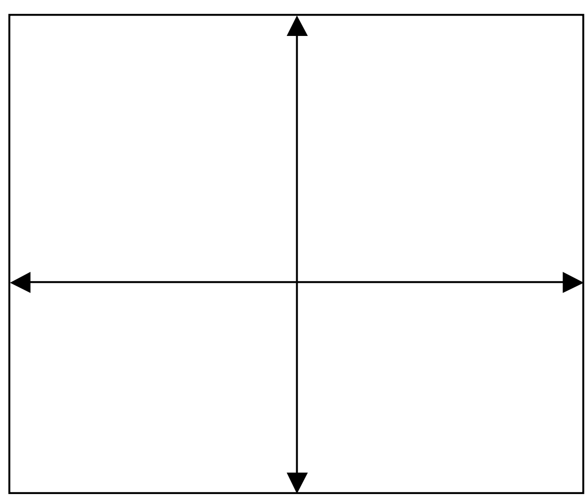


Determine whether the function has a vertical asymptote, or a removable discontinuity at $x = -1$. Graph the function to confirm your result.

Ex.8 $f(x) = \dfrac{x^2 - 2x - 8}{x + 1}$



Ex.9 $f(x) = \dfrac{x^2 + 1}{x + 1}$



THEOREM 1.15 Properties of Infinite Limits

Let c and L be real numbers and let f and g be functions such that

$$\lim_{x\to c} f(x) = \infty \qquad \text{and} \qquad \lim_{x\to c} g(x) = L.$$

1. Sum or difference: $\lim_{x\to c} [f(x) \pm g(x)] = \infty$

2. Product: $\lim_{x\to c} [f(x)g(x)] = \infty, \quad L > 0$

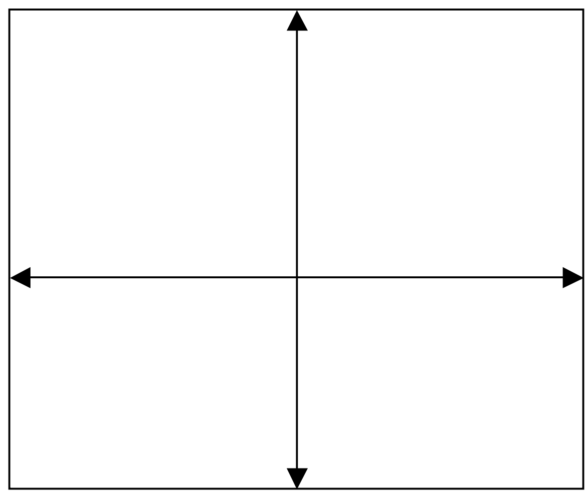
$\lim_{x\to c} [f(x)g(x)] = -\infty, \quad L < 0$

3. Quotient: $\lim_{x\to c} \dfrac{g(x)}{f(x)} = 0$

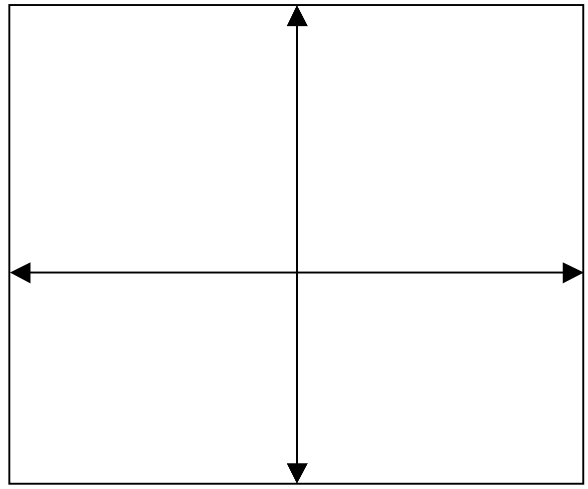
Similar properties hold for one-sided limits and for functions for which the limit of $f(x)$ as x approaches c is $-\infty$.

Find the one-sided limit. If it does not exist, explain why.

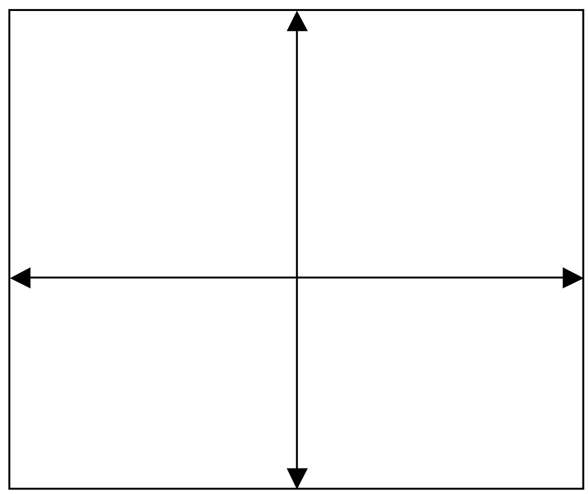
Ex.10 $\displaystyle\lim_{x\to -\frac{1}{2}^+} \frac{6x^2+x-1}{4x^2-4x-3}$



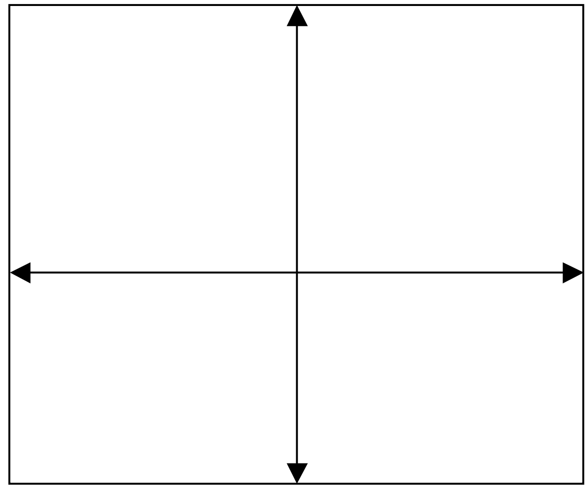
Ex.11 $\lim_{x \to 0^+}\left(6+\frac{1}{x^3}\right)$



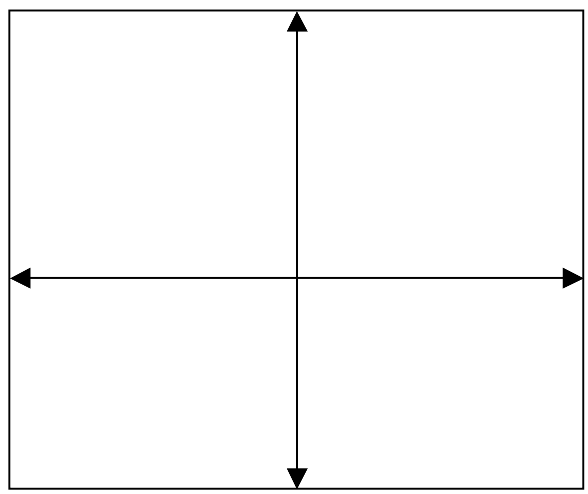
Ex.12 $\lim_{x \to 3^+}\left(\frac{x}{3}+\cot\left(\frac{\pi x}{2}\right)\right)$



Ex.13 $\lim_{x \to \frac{\pi}{2}^{+}} \frac{-2}{\cos(x)}$



Ex.14 $\lim_{x \to 0^{-}} \frac{x+2}{\cot(x)}$



Ex.15 $\lim_{x\to\frac{1}{2}^+} x^2\tan(\pi x)$